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Robustness of Hurst Exponent Estimates from BOLD fMRI Time Series Towards Additive Confounders

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Introduction

In the last decade, it has been shown that the Hurst exponent, H, of BOLD fMRI time series may carry relevant information regarding the state of the emitting voxel (see, for instance, [1], [2] and [3]). More precisely, it has been suggested to model resting-state time series as fractional Gaussian noise. Fractional Gaussian noise is a static process which can have very high auto-correlation at long time-separations, leading to what is called $1/f$ noise. The Hurst exponent is restricted to the interval $[0, 1]$, and characterizes the autocorrelation, with the high and low ends being completely correlated and anticorrelated, respectively, and $H = 0.5$ being regular white noise (no correlation). However, questions have been raised as to whether or not the fractal picture, which lies behind the Hurst exponent, can even be reliably used on time series of the length and quality relevant for BOLD fMRI measurements.
On this background, we have generated a large number of fGn-traces, and added various so-called confounders to these (see the corresponding box for a description of the confounders). The aim has then been to gauge what influence this has on the estimation of H.

Methods

The basis of this study is that the signal variance stemming from neural activity (thus excluding heart-beat, breathing, scanner noise and so forth) is fractionally Gaussian (fG) in nature. Thus it makes sense to test how this fG-quality is affected when confounders are added. We did this by generating artificial time series with predefined H-values in the interval $[.4, .6]$ (chosen because this is the range in which [1] and [3] measured H-values in), and tested how the H-estimates were related to the true values, and how this relationship was affected by the confounders. The time series were generated using a slightly modified version of the `ffgn.m` script from [4]. This script uses circular embedding of the covariance matrix at $H > 0.5$, and Lowen's method for $H < 0.5$; these methods are described in [5] and [6], respectively. Both these methods produce exact fGn-traces.
The H-estimates were produced using both the wavelet-based estimator of [7] and the function `wfbmesti` supplied with Matlab® (a description can be found at the Mathworks™ webpage).

Confounder Details

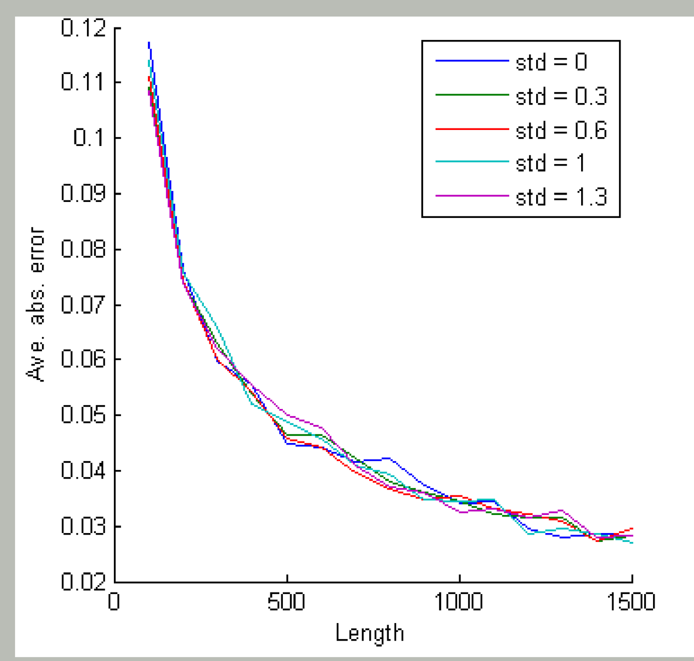
Below is a list of the different types of confounders used, and a description of each of them. For the two types of oscillation, the amplitude was **0.1** (to be compared with the std. of the fGn at **1.5**).

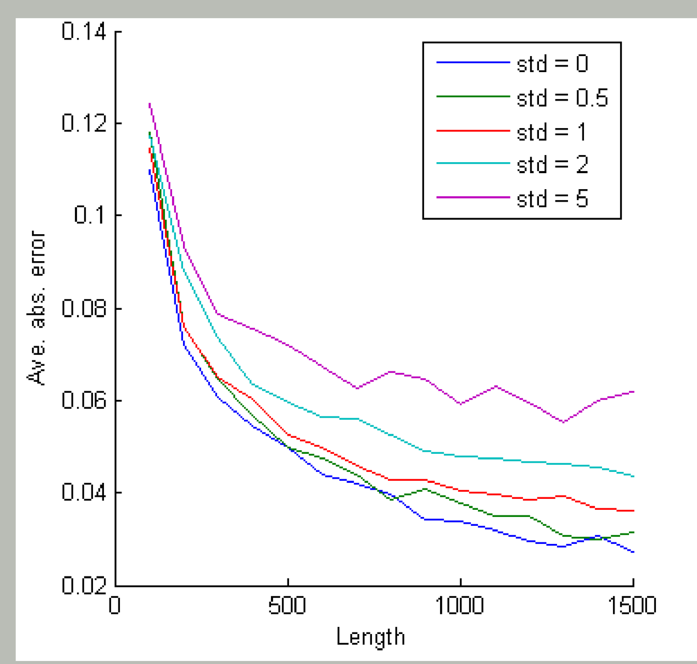
Name	Description
Drift	A first-order polynomial. In this case the strength-parameter that was varied was the inclination. An example of this could be a scanner-instability.
Chirp	A sinusoid with frequency varying gradually from one value to another. The parameters varied were the beginning and end frequency. This could, for instance, be the heart beat of a patient initially frightened of being in the scanner, but gradually relaxing.
White Noise	Gaussian-distributed white noise, with varying standard deviation. The mean was kept zero. One possible source of this could be the noise in the receptor-coil, but there are numerous possibilities.
Oscillation w. varying freq.	A signal of type $S(t_i) = A\sin(\omega_i t_i)$, in which t_i is an increasing series with constant step, and $\omega_i \sim\sim N(\omega, \sigma)$. The parameters varied were ω and σ.
Permanent shifts	To the last half of the time series was added a constant function. The strength-parameter of this approach was the value of the the constant function (the "size" of the shift).
Dot-shifts	At random points along the time series was added a constant. The parameters for this approach were the number of points (calculated as a percentage of the total number of points) and the size of the shift (the constant added). This was designed to mimic the effect of motion in a scanner that applies motion-correction on the fly (PACE).

Results

Below is shown graphical representations of some of the results. Interestingly enough, it appeared that with the exception of shifts and large-amplitude white noise, the most limiting factor towards getting reliable, correct, estimates is the length of the time series itself. Also, the bias introduced, even for small time series, is negligible. Thus, what is plotted is the average value of the absolute error, not the error itself, since that was virtually unaffected.

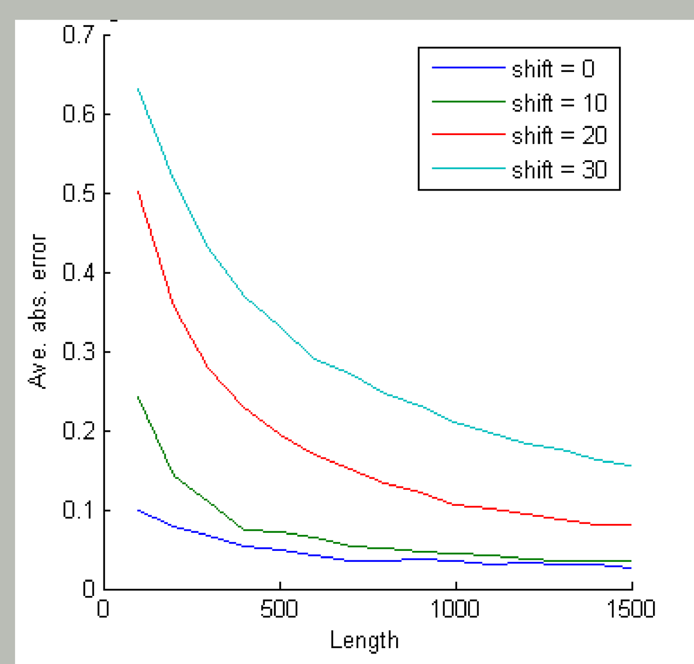
The effect when a sinusoid with varying frequency was added. It is clear that the confounder has no influence on the estimation of H, but rather that this only depends on the length of the time series. The figure legend refers to the normal distribution of the frequencies, as described in the box "Confounder Details".





In this figure, Gaussian white noise with mean zero was added to the time series. The legend refers to the distribution of this noise.

The result obtained from adding a constant shift to the time series. "Shift" is the difference in mean between the first and second halfs of the resulting time series.



We found that output 1 and 2 from `wfbmesti` perform the best (their average errors are indistinguishable). All data presented here is based on output 1.

Conclusion

It is assumed that H-values range between 0.4 and 0.7, including healthy and sick subjects (Maxim, (2005)). Requiring the uncertainty to be no more than 10 % of the full range implies that the absolute error has to be 0.03 or less. Thus, our results show that at least 1000 points are needed for a reliable estimate of H. Since non-movement-related noise-sources seem to have a limited effect on the H-estimation, a way to proceed might be to perform the scan with a small voxel-size, and then use several voxels when estimating H. If this method works, or time series much longer than 1000 points can be obtained, then H can be reliably used to describe BOLD fMRI time series. Of course, all this requires effective movement-removal.

Literature

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